

MATH 1113 – Mathematics for General Education
Exam 3

Definitions:

i) The number of **permutations** of n distinct objects selected k at a time is given by

$$P(n, k) = \frac{n!}{(n - k)!}$$

ii) The number of distinguishable **permutations** of n objects of r different types, where k_1 identical objects are of one type, k_2 of another type, k_3 of a third type, and so on, is given by

$$\frac{n!}{k_1! \times k_2! \times k_3! \cdots \times k_r!}$$

where $k_1 + k_2 + k_3 + \cdots + k_r = n$.

iii) The number of **combinations** of n objects selected k at a time is given by

$$C(n, k) = \frac{n!}{k! \times (n - k)!}$$

Instructions: Circle your final answers. Show all work. You may use a pocket calculator. No graphing or scientific calculators and NO SMARTPHONES.

1. A coin is tossed 3 times.

a.) How many elements are in the sample space associated with this experiment?

b.) List the elements in the sample space (use H for heads and T for tails). That is, write all possible outcomes of this experiment as elements of a set. (Hint: Make sure there are the same number of elements that you found in part a.)

c.) How many elements are in the event that exactly two of the three tosses land on tails?

d.) What is the probability that exactly two of the tosses land on tails?

e.) What are the odds in favor of the event that exactly two of the tosses land on tails?

2. Four-digit numbers are formed from the digits 1 through 8.

a.) How many four-digit numbers can be formed if this experiment is performed **with replacement**?

b.) How many four-digit numbers can be formed if this experiment is performed **without replacement**?

3. Consider an experiment in which an even number is chosen from the set $\{1, 2, 3, 4, \dots, 20\}$ and a six-sided die is rolled once. How many possible outcomes does this experiment have?

4. Twelve identical six-sided dice are rolled and the outcome is observed.

a.) How many elements are in the sample space associated with this experiment? (You may leave your answer as an exponent.)

b.) In how many ways can each of the numbers on the die (1 through 6) be rolled exactly twice? That is, how many elements are in the event that the outcome contains two 1's, two 2's, two 3's, two 4's, two 5's, and two 6's? You must write your answer as an exact number (no factorials remaining).

c.) What is the probability that the event from part b occurs?

5. Ten fair coins are tossed.

a.) How many elements are in the sample space associated with this experiment?

b.) What is the probability that at least one of the coins lands on tails? (Hint: You may want to use the complement rule.)

6. A single card is drawn from a standard deck.

a.) What is the probability that a Heart is drawn?

b.) What is the probability that a King is drawn?

c.) What is the probability that a Heart OR a King is drawn?

d.) What is the probability that a King OR a Queen is drawn?

7. Two cards are drawn from a standard deck.

a.) How many two-card hands are possible?

b.) In how many ways can two Clubs be drawn?

c.) What is the probability that two Clubs are drawn?